Quadratic Stochastic Operators with Infinite State Space

Nasir Ganikhodjaev
Institute of Mathematics, Tashkent, Uzbekistan,

Quadratic stochastic operator (in short QSO) was first introduced by Bernstein. The QSO was considered as important source of analysis for the study of dynamical properties and modelling in various fields such as biology, physics, game theory, control system. Such operator frequently arises in many models of mathematical genetics. Regularity, ergodicity and chaotic dynamics of QSO with finite state space were studied in a lot of papers.
We will consider QSO with infinite state space. Let $(X, \mathbb{F})$ be a measurable space and $S(X, \mathbb{F})$ be the set of all probability measures on $(X, \mathbb{F})$, where X is a state space and $\mathbb{F}$ is σ-algebra of subsets of X.
Let $\{P(x, y, A) : x, y \in X, A \in \mathbb{F}\}$ be a family of functions on $X \times X \times \mathbb{F}$ that satisfy the following conditions:
i) $P(x, y, \cdot) \in S(X, \mathbb{F})$, for any fixed $x, y \in X$, that is, $P(x, y, \cdot) : \mathbb{F} \to [0, 1]$ is the probability measure on $\mathbb{F}$;
ii) $P(x, y, A)$ regarded as a function of two variables x and y with fixed $A \in \mathbb{F}$ is measurable function on $(X \times X, \mathbb{F} \otimes \mathbb{F})$;
iii) $P(x, y, A) = P(y, x, A)$ for any $x, y \in X, A \in \mathbb{F}$.
We consider a non-linear transformation (QSO) $V : S(X, \mathbb{F}) \to S(X, \mathbb{F})$ defined by

$$(V\lambda)(A) = \int_X \int_X P(x, y, A) d\lambda(x) d\lambda(x),$$

where $\lambda \in S(X, \mathbb{F})$ is an arbitrary initial probability measure and $A \in \mathbb{F}$ is an arbitrary measurable set.
Let $\xi = \{A_1, A_2, \cdots, A_m\}$ be a measurable $m-$partition of the set X and $\zeta = \{B_{ij} : i, j = 1, 2, \cdots, m\}$ be a corresponding partition of the Cartesian square of $X \times X$, where $B_{ii} = A_i \times A_i$ for $i = 1, 2, \cdots, m$ and $B_{ij} = (A_i \times A_j) \cup (A_j \times A_i)$ if $i \neq j$.
We select a family $\{\mu_{ij} : i, j = 1, \cdots, m\}$ of probability measures on $(X, \mathbb{F})$ and define probability measure $P(x, y, A)$ as follows:

$$P(x, y, A) = \mu_{ij}(A) \text{ if } (x, y) \in B_{ij}, \quad i, j = 1, \cdots, m$$

for arbitrary $A \in \mathbb{F}$.
Then the QSO constructed by this family of functions one can consider as approximation of QSO V by QSO with finite state space.
For several years now, joint research has been carried out with colleagues from IIUM to study such operators.
The purpose of this presentation is to review the results obtained earlier and to formulate new problems.